Solving Equations

Solving linear equations is just a matter of undoing operations that are being done to the variable. The task is always to isolate the variable -- get the variable ALONE on one side of the equal sign.

Remember when solving equations to "keep the equation balanced" by making the same changes to BOTH sides of the equal sign.

Example 1: In a simple equation, you may only have to undo one operation to solve the equation.

Solve this equation for *x*: $x + 3 = 8$	
The variable is *x* and we need to get it alone. In the problem, 3 is being added to the variable, so to get rid of the added 3, we do the opposite ---- subtract 3. We are actually employing the additive inverse property to create a 0 since +3 - 3 = 0. Then the additive identity is used to get x alone since x + 0 = x. (Remember to subtract 3 from **both** sides of the equation to **"keep the equation balanced".**)	$x + 3 = 8$ $-3 \quad -3$ $x \quad = 5$
Check your answer: You will always know if your answer is correct by doing a simple "check" -- substitute your answer into the original equation and see if the result is true.	**Check:** $x + 3 = 8$ $5 + 3 = 8$ $8 = 8$ **true**

Example 2: In an equation which has more than one operation, we have to **undo the operations in the correct order**. First, undo addition or subtraction, then undo multiplication or division.

Solve this equation for *x*: $5x - 2 = 13$	
The variable is *x*. The question is multiplying *x* by 5, and then subtracting 2. First, undo the subtraction by adding 2. Then, undo the multiplication by dividing by 5. This process is actually employing the multiplicative inverse to create the value of 1 and then employing the multiplicative identity to isolate the x. (Remember to perform your changes to **both** sides of the equation to **"keep the equation balanced".**)	$5x - 2 = 13$ $+2 \quad +2$ $5x = 15$ $\frac{5x}{5} = \frac{15}{5}$ $x = 3$
Check your answer:	**Check:** $5x - 2 = 13$ $5(3) - 2 = 13$ $15 - 2 = 13$ $13 = 13$ **true**

Example 3: Suppose there are variables on both sides of the equation. The trick now, is to get the variables on the **same** side by adding them or subtracting them.

Solve this equation for *x*: $4x + 5 = x - 4$

This question has two terms with the variable; 4*x* and *x*. We need to get the variables combined into one term.

Move the variable with the smaller coefficient, namely *x*. The sign in front of the *x* is implied to be +.

Subtract *x* from both sides.

$$4x + 5 = x - 4$$
$$-x \qquad -x$$
$$3x + 5 = -4$$

Now we proceed as before.

$$3x + 5 = -4$$
$$-5 \qquad -5$$
$$3x = -9$$
$$\frac{3x}{3} = \frac{-9}{3}$$
$$x = -3$$

(Remember to perform your changes to **both** sides of the equation to **"keep the equation balanced".**)

Check your answer:	**Check:** $4x + 5 = x - 4$ $4(-3) + 5 = -3 - 4$ $-12 + 5 = -7$ $-7 = -7$ **true**
Hint: Some students think of "moving" one variable to the other side of the equal sign as "moving" the variable over the "equal sign bridge". Moving any term across the "equal sign bridge" changes the term's sign (like paying a toll). $4x + 5 = x - 4$ $4x - x + 5 = -4$ as the x moves to the left over the "equal sign bridge", it changes its sign to negative.	

Example 4: Sometimes there are equations which have multiple terms on the **same side**. The trick here is to combine all the similar terms before solving.

Solve this equation for *y*: $7y + 5 - 3y + 1 = 2y + 2$

First combine the similar terms on the left side.
(Don't forget to take the sign in front of the term. If there isn't a sign in front of the term, it is considered +.)
Combining: $7y - 3y = 4y$
$+5$ and $+1 = +6$

Now proceed as before.

(Remember to perform your changes to **both** sides of the equation to **"keep the equation balanced".**)

$$7y + 5 - 3y + 1 = 2y + 2$$

$$4y + 6 = 2y + 2$$
$$-2y \qquad -2y$$
$$2y + 6 = 2$$
$$-6 \qquad -6$$
$$\frac{2y}{2} = \frac{-4}{2}$$
$$y = -2$$

Check your answer:

Check:
$7y + 5 - 3y + 1 = 2y + 2$
$7(-2) + 5 - 3(-2) + 1 = 2(-2) + 2$
$-14 + 5 + 6 + 1 = -4 + 2$
$-2 = -2$ **true**

Example 5: There are also equations with parentheses. The first step in these problems is to multiply and get rid of the parentheses.

Solve this equation for *n*: $3(n - 1.8) = 2n + 1$

First distribute the 3 -- multiply through the parentheses by 3.

$$3(n - 1.8) = 2n + 1$$
$$3n - 5.4 = 2n + 1$$

Now proceed as normal.

$$3n - 5.4 = 2n + 1$$
$$-2n \qquad -2n$$
$$n - 5.4 = 1$$
$$+ 5.4 \qquad + 5.4$$
$$n = 6.4$$

(Remember to perform your changes to **both** sides of the equation to **"keep the equation balanced".**)

Check your answer:	**Check:** $3(n - 1.8) = 2n + 1$ $3(6.4 - 1.8) = 2(6.4) + 1$ $3(4.6) = 12.8 + 1$ $13.8 = 13.8$ **true**

Example 6: The last type of equation contains fractions.

Solve for *x*: (method 1)

$$2x + \frac{1}{3} = 2$$

First subtract the fraction from both sides. Use may wish to use your calculator to do the arithmetic.

$$2x + \frac{1}{3} = 2$$
$$-\frac{1}{3} \quad -\frac{1}{3}$$
$$2x = 1\tfrac{2}{3}$$

Now divide both sides by 2. Again, use your calculator to check your arithmetic.

$$\frac{2x}{2} = \frac{1\tfrac{2}{3}}{2}$$

$$x = \frac{5}{6}$$

Check your answer:	**Check:** $2(5/6) + (1/3) = 2$ $(10/6) + (1/3) = 2$ $(5/3) + (1/3) = 2$ $6/3 = 2$ $2 = 2$ **true**